Dynamic Semantics Meets Cognitive and Social Science

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1 Introduction

Cognitive Science reveals the importance of how to cognize agents' environments from a "private" point of view for modeling agents. This point has been urging linguists to develop "cognitive linguistics"[38], "cognitive semantics"[27,36, 37], or "cognitive grammar"[28,29]. As a matter of fact, cognitive semanticists find and explain linguistic expressions that reflect agents' "private" cognitions and need an explanatory device involving agents' "private" cognitions. However, this movement reminds us of the notorious philosophical debate "Language is private or public?". If language is purely private, we cannot explain why we use linguistic expressions to communicate with the others. From a point of view of communication, not only language but also how to interpret the expressions and what the expressions means must be public or social. Therefore, simplistically to say, Cognitive Science and Social Science conflict when they handle language. But both points actually reflect the nature of language.

In this paper, I regard a linguistic expression as an action of communicating information on each agent's "private" cognition, or instruction of how to cognize agents' shared environment. That is, language can be regarded as not only action of private cognition but also action instructing the other agents on private experiences in order to inform the other agents the private cognition. We will call this idea as "language-as-cognitive-instruction". This claim presupposes that each agent empathizes the other agents in their society (or community). Furthermore, agents must have sharable formal devices to empathize a private cognition. I will model this device as the *Private Viewpoint Model*, and language as a social action operating states of the Private Viewpoint Model. By adopting Private Viewpoint Models and the idea "language-as-cognitive-instuction", my proposed theory will have its conceptual basis on a theory similar to the Gibsonian or ecological psychology [11, 30, 34] rather than the Gestalt psychology that is adopted by coginitive linguistics/semantics (See [38]). This point means that cognitive linguists focus on what is cognized, but my proposed theory focuses on how to cognize things and how to share it among agents. Both theories handle the privateness of cognition, but by these points both are distinguished.

The idea of Private Viewpoint Models and "language-as-cognitive-insturction" is compatible with a framework of formal semantics, *Dynamic Semantics* of language, programs or actions. Therefore, I will propose Dynamic Logic of Motions from Private Viewpoints, Dynamic Logic of Cogintion, and Dynamic Logic of Empathy, based on this idea.

2 A Short Introduction to Dynamic Semantics

Dynamic Semantics is a framework of semantics of logical languages developed by [3, 35, 13, 31, 14, 15, 40, 39, 23, 46, 4, 47, 41, 26, 32] and others, exploiting Heim's [19] idea of *File Change Semantics* of English. Theories of Dynamic Semantics can be classified into at least following classes[1]:

	Literatures	Langaguage	Model	Interpretation
Dynamic Interpretations	[3, 35, 14, 23]	$L_Q(\Phi)$	$W = \{g \| g : Var \to D\}$	$[\![,]\!]$
Extensions of Dynamic Logics	[45, 10, 9, 2, 44]	$L(\Phi, \Pi)$	$\langle W, \langle [\![\pi]\!] \rangle_{\pi \in Prog(\Phi,\Pi)}, V \rangle$	$\models$
Dynamic Modal Logics	[39, 5, 41]	$L(\Phi, \Pi, \delta, \sigma)$	$\langle W, \sqsubseteq, V \rangle$	$\models, [\![,]\!]$
Dynamic Intensional Logics	[13, 31, 4]	$\lambda \to$	D^∞	$(\lambda \to) \to D^\infty$

where Φ is a set of atomic sentences, Π a set of atomic actions, L_Q the language of first-order logic with generalized quantifiers, $Prog(\Phi, \Pi)$ the actions generated by Φ and Π, $V : \Phi \to pow(W)$ (valuation), $\delta : L(\Phi, \Pi, \delta, \sigma) \to Prog(\Phi, \Pi, \delta, \sigma,$ $\sigma : Prog(\Phi, \Pi, \delta, \sigma) \to L(\Phi, \Pi, \delta, \sigma)$, $\models \subseteq (Model \times W) \times L(\Phi, \ldots)$, $[\![,]\!] \subseteq (Model \times W) \times Prog(\Phi, \Pi, \ldots) \times (Model \times W)$, $\lambda \to$ the language of the simple type theory, and D^∞ is the solution of $D \cong (D \to D)$, i.e., the Scott domain. For the detail of languages, we will see later. For Dynamic Logics, see [16, 17, 12, 25, 18].

The point of theories of Dynamic Semantics is that they interpret expressions as state transitions. The most basic and logically refined framework of Dynamic Semantics is Groenendijk and Stokhoff [14]'s *Dynamic Predicate Logic (DPL).* To compare DPL with the Tarskian static semantics, firstly I introduce the language and semantics of the first-order logic with binary generalized quantifiers, d as follows:

Definition 1. *Let* $Var = \{x_1, \ldots, x_n\}$ *be a set of individual variables and* $\mathcal{M} = \langle D^{\mathcal{M}}, \langle R_i^{\mathcal{M}} \rangle_{i \in I}, \langle q_j^{\mathcal{M}} \rangle_{j \in J} \rangle$ *be a model of a first order langauge with binary generalized quantifiers, where* $D^{\mathcal{M}}$ *is the domain of* $\mathcal{M}$*, a set of individuals,* $R_i^{\mathcal{M}} \subseteq D^{\mathcal{M}\rho(i)}$ *(*$\rho(i)$ *is the arity of* R*), a relation on* $D^{\mathcal{M}}$*, and* $q_j^{\mathcal{M}} \subseteq$

[1] Kamp[21, 22]'s *Discourse Representation Theory (DRT)* and Gabbay[7]'s *Labelled Deductive Systems (LDS)* can also be considered as Dynamic Semantics, but in the sense that dynamic semantics regards meaning as state transitions, they are not dynamic semantics, but rather *Procedural Semantics* as well as Hintikka[20]'s *Game-Theoretical Semantics*, since they are based on semantic tableau-like reduction procedures. See van Benthem and van Eijck [43] for Procedural Semantics. However, the recent development of Dynamic Semantics seems to focus on the relation between Dynamic Logics and Game Logics including knowledge sharing processes among communicative agents. See [33, 42] for the survey of this movement.

$pow(D^{\mathcal{M}}) \times pow(D^{\mathcal{M}})$, a binary generalized quantifier on $D^{\mathcal{M}}$. A variable assignment g that is a function from Var to $D^{\mathcal{M}}$.

The syntax and semantics of $\varphi \in L_Q(\Phi)$ is defined as follows:

$$\varphi ::= R(x_1, \ldots, x_n) | \varphi_1 \wedge \varphi_2 | \varphi_1 \rightarrow \varphi_2 | \exists x \varphi | \forall x \varphi | Qx(\varphi_1, \varphi_2) | \bot$$

where Q is a binary generalized quantifier symbol, and

- $\mathcal{M}, g \models R(x_1, \ldots, x_n)$ *iff* $\langle g(x_1), \ldots, g(x_n) \rangle \in R^{\mathcal{M}}$
- $\mathcal{M}, g \models \varphi_1 \wedge \varphi_2$ *iff* $\mathcal{M}, g \models \varphi_1$ *and* $\mathcal{M}, g \models \varphi_2$
- $\mathcal{M}, g \models \varphi_1 \rightarrow \varphi_2$ *iff* $\mathcal{M}, g \models \varphi_1$ *implies* $\mathcal{M}, g \models \varphi_2$
- $\mathcal{M}, g \models \exists x \varphi$ *iff* $\mathcal{M}, g[e/x] \models \varphi$, *for some* $e \in D^{\mathcal{M}}$.
- $\mathcal{M}, g \models \forall x \varphi$ *iff* $\mathcal{M}, g[e/x] \models \varphi$, *for all* $e \in D^{\mathcal{M}}$.
- $\mathcal{M}, g \models Qx(\varphi_1, \varphi_2)$ *iff* $\langle \{e | \mathcal{M}, g[e/x] \models \varphi_1\}, \{e | \mathcal{M}, g[e/x] \models \varphi_2\} \in q^{\mathcal{M}}$.
- $\mathcal{M}, g \not\models \bot$

On the other hand, DPL gives relational truth conditions such that formula φ changes g to h in $\mathcal{M}$, written $g \llbracket \varphi \rrbracket^{\mathcal{M}} h$, as follows.

Definition 2.

- $g \llbracket R(x_1, \ldots, x_n) \rrbracket^{\mathcal{M}} h$ *iff* $g = h$ *and* $\langle g(x_1, \ldots, g(x_n)) \rangle \in R^{\mathcal{M}}$
- $g \llbracket \varphi_1 \wedge \varphi_2 \rrbracket^{\mathcal{M}} h$ *iff for some* i, $g \llbracket \varphi_1 \rrbracket^{\mathcal{M}} i$ *and* $i \llbracket \varphi_2 \rrbracket^{\mathcal{M}} h$
- $g \llbracket \varphi_1 \rightarrow \varphi_2 \rrbracket^{\mathcal{M}} h$ *iff* $g = h$ *and*
  for all i, $g \llbracket \varphi_1 \rrbracket^{\mathcal{M}} i$ *implies that for some* j, $i \llbracket \varphi_2 \rrbracket^{\mathcal{M}} j$
- $g \llbracket \exists x \varphi \rrbracket^{\mathcal{M}} h$ *iff* $g[e/x] \llbracket \varphi \rrbracket^{\mathcal{M}} h$ *for some* $e \in D^{\mathcal{M}}$,
- $g \llbracket \forall x \varphi \rrbracket^{\mathcal{M}} h$ *iff* $g = h$ *and for all* $e \in D^{\mathcal{M}}$, $i = g[e/x]$ *implies that for some* j, $i \llbracket \varphi \rrbracket^{\mathcal{M}} j$.
- $g \llbracket Qx(\varphi_1, \varphi_2) \rrbracket^{\mathcal{M}} h$ *iff*[2] $g = h$ *and* $\langle \{i(x) | g \llbracket \varphi_1 \rrbracket^{\mathcal{M}} i\}, \{j(x) | g \llbracket \varphi_1 \wedge \varphi_2 \rrbracket^{\mathcal{M}} j\} \rangle \in q^{\mathcal{M}}$
- $\langle g, h \rangle \notin \llbracket \bot \rrbracket^{\mathcal{M}}$ *for any* g, h

As a result, the following conditions (1a,b) that express the logical properties of anaphora as in (1c,d) respectively hold in DPL:

(1) a. $\llbracket (\exists x \varphi_1) \rightarrow \varphi_2 \rrbracket^{\mathcal{M}} = \llbracket \forall x (\varphi_1 \rightarrow \varphi_2) \rrbracket^{\mathcal{M}}$
    b. $\llbracket (\exists x \varphi_1) \wedge \varphi_2 \rrbracket^{\mathcal{M}} = \llbracket \exists x (\varphi_1 \wedge \varphi_2) \rrbracket^{\mathcal{M}}$
    c. "If a farmer owns a donkey, he beats it." = "Every farmer who owns a donkey beats it."
    d. "A farmer owns a donkey. He beats it." = "A farmer who owns a donkey beats it."

On the other hand, these conditions are not valid in the static semantics of $L_Q(\Phi)$ as in defition 1.

Dynamic Logics are modal logics of which modal operators express programs or actions. In particular, *Propositional Dynamic Logic (PDL)* is the most basic dynamic logic, defined as follows.

[2] This is the weak reading. The strong reading is defined as follows:
$g \llbracket Qx(\varphi_1, \varphi_2) \rrbracket^{\mathcal{M}} h$ iff $g = h$ and $\langle \{i(x) | g \llbracket \varphi_1 \rrbracket^{\mathcal{M}} i\}, \{j(x) | g \llbracket \varphi_1 \rightarrow \varphi_2 \rrbracket^{\mathcal{M}} j\} \rangle \in q^{\mathcal{M}}$.
See [4, 24] for these distinctions.

Definition 3. *Let $p \in \Phi$ be an atomic sentences and $\alpha \in \Pi$ be an atomic program or action. Sentence $\varphi \in L(\Phi, \Pi)$ and program $\pi \in Prog(\Phi, \Pi)$ of PDL are defined by the following BNF grammar:*

$$\varphi ::= p | \varphi_1 \rightarrow \varphi_2 | [\pi]\varphi | \bot$$

$$\begin{aligned} \pi ::= \ & \alpha \\ & |\varphi? && \textit{(rich test)} \\ & |\pi_1 \cup \pi_2 && \textit{(nondeterministic choice)} \\ & |\pi_1 ; \pi_2 && \textit{(sequential composition)} \\ & |\pi^* && \textit{(iteration)} \end{aligned}$$

Let $\mathcal{M} = \langle W, \langle [\![\pi]\!]^{\mathcal{M}} \rangle_{\pi \in Prog(\Phi, \Pi)}, V \rangle$ be a Kripke model. Then the semantics of φ, i.e., relation $\mathcal{M}, w \models \varphi$ is defined as in definition 1 by adding the following condition on modal formula:

$$\mathcal{M}, w \models [\pi]\varphi \textit{ iff for all } u, \ w [\![\pi]\!]^{\mathcal{M}} u \textit{ implies } \mathcal{M}, u \models \varphi.$$

This condition means that at state w if we execute program π, then condition φ holds at the resulting state u). The semantics of programs,i.e., $[\![\pi]\!]^{\mathcal{M}}$, is defined as follows:

$$\begin{aligned} & w [\![\varphi?]\!]^{\mathcal{M}} u \textit{ iff } u = w \textit{ and } \mathcal{M}, u \models \varphi \\ & w [\![\pi_1 ; \pi_2]\!]^{\mathcal{M}} u \textit{ iff there is } v \textit{ such that } w [\![\pi_1]\!]^{\mathcal{M}} v \textit{ and } v [\![\pi_2]\!]^{\mathcal{M}} u \\ & w [\![\pi_1 \cup \pi_2]\!]^{\mathcal{M}} u \textit{ iff either } w [\![\pi_1]\!]^{\mathcal{M}} u \textit{ or } w [\![\pi_2]\!]^{\mathcal{M}} u \\ & w [\![\pi^*]\!]^{\mathcal{M}} u \textit{ iff } \langle w, u \rangle \in \bigcup_{n \geq 0} [\![\pi^n]\!]^{\mathcal{M}}, \textit{ where } \pi^0 = \top? \textit{ and } \pi^{n+1} = \pi ; \pi^n. \end{aligned}$$

Usual programs can be defined as follows:

$$\mathsf{skip} \equiv \top? \qquad \mathsf{fail} \equiv \mathsf{abort} \equiv \bot?$$

$$(\mathsf{if}\ \varphi\ \mathsf{then}\ \pi_1\ \mathsf{else}\ \pi_2) \equiv ((\varphi?; \pi_1) \cup (\neg\varphi?; \pi_2))$$

$$(\mathsf{while}\ \varphi\ \mathsf{do}\ \pi) \equiv ((\neg\varphi?; \pi)^*; \varphi?)$$

If $W = \{w | w : Var \rightarrow D^{\mathcal{M}}\}$, where $D^{\mathcal{M}}$ is a set of individuals, and α is either $\mathsf{x} := \tau$ or $\mathsf{x} :=?$, and new formula $\forall \mathsf{x} \varphi$ is added

$$\begin{aligned} & w [\![\mathsf{x} := \tau]\!]^{\mathcal{M}} u \text{ iff } u = w[\tau_w^{\mathcal{M}} / x] \\ & w [\![\mathsf{x} :=?]\!]^{\mathcal{M}} u \text{ iff } u = w[e/x] \text{ for some } e \in D^{\mathcal{M}} \end{aligned}$$

then the language $L(\Phi, \Pi)$ is called the language of *First Order Dynamic Logic* (DL). The language of PDL can be translated into the programs of DL as follows ([14]):

$$\begin{aligned} & \dagger R(x_1, \ldots, x_n) = R(x_1, \ldots, x_n)? \\ & \dagger\bot = \bot? \\ & \dagger\varphi_1 \wedge \varphi_2 = (\dagger\varphi_1; \dagger\varphi_2) \\ & \dagger\varphi_1 \rightarrow \varphi_2 = ([\dagger\varphi_1]\neg[\dagger\varphi_2]\bot?) \\ & \dagger\exists x\varphi = (x :=?; \dagger\varphi) \\ & \dagger\forall x\varphi = ([x :=?]\neg[\dagger\varphi]\bot?) \end{aligned}$$

Therefore, PDL can be considered as a fragment of DL.

3 Dynamic Logic of Motion from Private Viewpoints and Cognition

The *Private Viewpoint Model* is a realistic theory of private cognition exploiting a kind of metaphor of photography (See [?] for a photographic metaphor in cognitive psychology.) It has an ontology based on events and other directed spatio-temporal entities and their relations. The Private Viewpoint Model assigns to such entities roles: *agent*, *viewfinder*, *focus*, *subject*, and *background*. These roles on the whole plays similar roles in Cognitive Semantics such as *trajector* (tr) and *landmark* (lm).

3.1 The Private Viewpoint Model

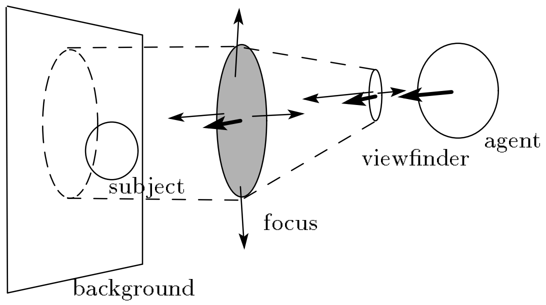


Fig. 1. Private Viewpoint Model

A state w is a tuple $\langle dom(w), g, R_i\rangle_{i\in I(w)}$, called a *Private Viewpoint Model*, where $dom(w)$ is a set of directed spatio-temporal entities such as a (agent), v (viewfinder), f (focus), b (background), s (subject), $\{R_i\}_{i\in I(w)} = \{\sqsubseteq, N, \mapsto\}$, and variable assginment g. They satisfy at least the following conditions:

1. $a \mapsto v$ (the agent looks the viewfinder)
2. $v \mapsto f$ (the viewfinder is directed toward the focus)
3. $\exists x.f \mapsto x \wedge x \sqsubseteq b$ (the focus is directed toward a part of the background)
4. $sOf \vee \neg\exists x.x = s$ (the subject overlaps with the focus, exists between the focus and the background, or not exist)
5. (the viewfinder moves toward or apart from the focus along the viewfinder's direction)
6. (the focus can move toward an arbitrary direction, but the direction is fixed toward the background)
7. $\neg vN_f b$ (the viewfinder is not closer to the background than the focus)
8. $\neg aN_v f$ (the agent is not closer to the focus than the viewfinder)
9. (change without pan, agent change ($a := x$), and background change ($b := x$) preserves the direction)

3.2 Dynamic Logic of Private Viewpoints (*DLPV*)

The language of $DLPV$, $L(Term, \Phi, \Pi)$ and its semantics is defined as the following BNF grammar:

$$\left.\begin{array}{ll}\tau ::= \mathsf{a} & \text{(agent)} \\ \quad|\mathsf{v} & \text{(viewfinder)} \\ \quad|\mathsf{f} & \text{(focus)} \\ \quad|\mathsf{b} & \text{(background)} \\ \quad|\mathsf{s} & \text{(subject)}\end{array}\right\} \text{(pointers)}$$

$$\begin{array}{ll}p ::= \tau_1 \sqsubseteq \tau_2 & (\tau_1 \text{ is included to } \tau_2) \\ \quad|\tau_1 \mathsf{N}_{\tau_3} \tau_2 & (\tau_1 \text{ is close to } \tau_2 \text{ more than } \tau_3) \\ \quad|\tau_1 \mapsto \tau_2 & (\tau_1 \text{ looks towards } \tau_2) \\ \quad|\mathsf{R}(\tau_1, \ldots, \tau_n)|\tau_1 = \tau_1 & \end{array}$$

and $(\tau_1 \sqsubset \tau_2) \equiv (\tau_1 \sqsubseteq \tau_2 \wedge \neg \tau_2 \sqsubseteq \tau_1)$ (proper inclusion), $(\tau_1 \mathsf{O} \tau_2) \equiv (\exists v(\tau_1 \sqsubseteq v \wedge \tau_2 \sqsubseteq v))$ (overlapping), $(\tau_1 \mathsf{S} \tau_2) \equiv (\neg \tau_1 \mathsf{O} \tau_2)$ (separation), $(\mathsf{InFocus}) \equiv (\mathsf{sOf})$, $(\mathsf{OutOfFocus}) \equiv (\mathsf{sSf})$,

$$\begin{array}{ll}\alpha ::= \mathsf{pan}\ r & \text{(pan the focus toward } r) \\ \quad|\mathsf{zoom\ in} & \text{(move the viewfinder toward the focus)} \\ \quad|\mathsf{zoom\ out} & \text{(move the viefinder apart from the focus)} \\ \quad|\mathsf{come} & \text{(move the focus with the subject toward the viewfinder)} \\ \quad|\mathsf{leave} & \text{(move the focus with the subject apart from the viewfinder)} \\ \quad|\mathsf{go}\ r' & \text{(move the focus with the subject toward } r) \\ \quad|x :=?_X & \text{(restricted random assignment)}\end{array}$$

where $r \in \{?, up, down, left, right\}$ and $r' \in \{?, up, down, left, right, forward, backward\}$.

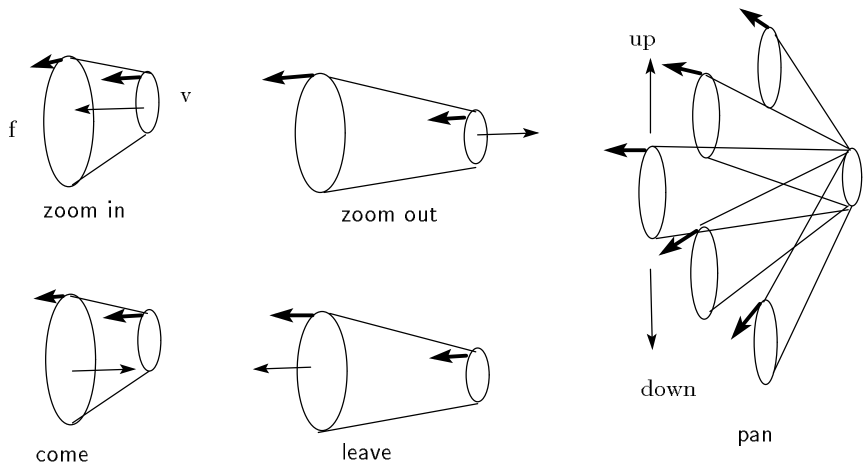


Fig. 2. Actions on the viewfinder and the focus

$w_1 [\![\mathsf{zoom\ in}]\!]^{\mathcal{M}} w_2$ iff $\mathsf{f}_{w_2} = \mathsf{f}_{w_1}$ and $\mathsf{v}_{w_2} N_{\mathsf{v}_{w_1}} \mathsf{f}_{w_2}$,

w_1 $[\![\mathsf{zoom\ out}]\!]^{\mathcal{M}} w_2$ iff $\mathsf{f}_{w_2} = \mathsf{f}_{w_1}$ and $\mathsf{v}_{w_1} N_{\mathsf{v}_{w_2}} \mathsf{f}_{w_2}$,
w_1 $[\![\mathsf{come}]\!]^{\mathcal{M}} w_2$ iff $\mathsf{v}_{w_2} = \mathsf{v}_{w_1}$ and $\mathsf{f}_{w_2} N_{\mathsf{f}_{w_1}} \mathsf{v}_{w_2}$,
w_1 $[\![\mathsf{leave}]\!]^{\mathcal{M}} w_2$ iff $\mathsf{v}_{w_2} = \mathsf{v}_{w_1}$ and $\mathsf{f}_{w_1} N_{\mathsf{f}_{w_2}} \mathsf{v}_{w_2}$,
w_1 $[\![\mathsf{pan}\ r]\!]^{\mathcal{M}} w_2$ iff $\mathsf{v}_{w_2} = \mathsf{v}_{w_1}$ and $\mathsf{f}_{w_2} r \mathsf{f}_{w_1}$,
w_1 $[\![\mathsf{pan}\ ?]\!]^{\mathcal{M}} w_2$ iff $\mathsf{v}_{w_2} = \mathsf{v}_{w_1}$ and for some r, $\mathsf{f}_{w_2} r \mathsf{f}_{w_1}$.
w_1 $[\![\mathsf{go}\ r']\!]^{\mathcal{M}} w_2$ iff $\mathsf{v}_{w_2} = \mathsf{v}_{w_1}$, $\mathsf{s}_{w_i} \sqsubset \mathsf{f}_{w_i}$ (for $i \in \{1,2\}$), $\mathsf{s}_{w_1} \approx \mathsf{s}_{w_2}$ and $\mathsf{f}_{w_2} r \mathsf{f}_{w_1}$, where $x \approx y$ iff x and y are stages of the same individual.
w_1 $[\![\mathsf{go}\ ?]\!]^{\mathcal{M}} w_2$ iff $\mathsf{v}_{w_2} = \mathsf{v}_{w_1}$, $\mathsf{s}_{w_i} \sqsubset \mathsf{f}_{w_i}$ (for $i \in \{1,2\}$) and for some r', $\mathsf{f}_{w_2} r' \mathsf{f}_{w_1}$.
w_1 $[\![x :=?_X]\!]^{\mathcal{M}} w_2$ iff $g_{w_2} = g_{w_1}[e/x]$ for some $e \in X^{\mathcal{M}}$.

Other actions can be defined as follows:

$$(\mathsf{find}\ \phi) \equiv (\mathsf{while}\ \neg\phi(\mathsf{s})\ \mathsf{do}\ \mathsf{s} :=?_{E(\mathsf{b})})$$

$$(\mathsf{pan\ to}\ \phi) \equiv (\mathsf{while}\ \neg\phi(\mathsf{s})\ \mathsf{do}\ (\mathsf{pan}\ \mathsf{s} :=?_{E(\mathsf{f})}))$$

where $E(x)$ is a set of entities live on x.

In the next subsections, I will apply *DLPV* to natural language semantics, comapring the approach of the Cognitive Semantics [27–29, 36, 37] and the Vector Space Semantics [48–51, 6][3].

3.3 Directional Prepositions

Directional prepositioins can be listed as follows:

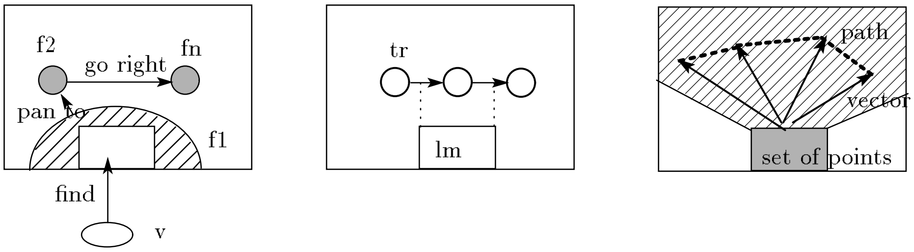
(2) a. Source prepositions: from, out of, off
b. Goal prepositions: to, into, onto
c. Route prepositions: through, across, along, around, over

[3] A *vector space* over the field of $\mathbb{R}$ is a quadruple $\langle V, 0, +, \cdot \rangle$, where V is a set, $0 \in V$ (the zero vector), commutative and associative function $+ : V \times V \to V$ (vector addition), and associative function $\cdot : \mathbb{R} \times V \to V$ (scalar multiplication), satisfying the following conditions:

$\forall v \in V.0 + v = v + 0 = v$
$\exists u \in V.v + u = 0$ (written $-v$)
$\forall s \in \mathbb{R} \forall u, v \in V.s(u + v) = su + sv$
$\forall s, r \in \mathbb{R} \forall u \in V.(s + r)v = sv + rv$
$\forall v \in \mathbb{R}.1v = v$

$|,| : V \to \mathbb{R}$ is called the *norm*. The domain of verctors Vec is $\bigcup_{w \in V} V_w$, where $V_w = \{(w, v) | v \in V\}$, and the domain of points Point is V. Vector $\mathbf{v}= (w, v)$ is a *boundary* vector of A, written $boundary(\mathbf{v}, A)$, iff w is in the boundary of A. $\mathbf{v}= (w, v)$ is a *closest* vector to A, written $closest(\mathbf{v}, A)$, iff for every vector $\mathbf{w}= (w_1, w_2)$ s.t. $boundary(\mathbf{w}, A)$ and $v = w_2$, $|\mathbf{v}| \leq |\mathbf{w}|$. If $v \in A$, $\mathbf{v}$ is internally closest to A, written $int(\mathbf{v}, A)$. Otherwise, $\mathbf{v}$ is externally closest to A, written $ext(\mathbf{v}, A)$.

They semantically include the notion of "paths". In coginitive linguistics, paths are conceived as "scanning" in image schemata, and in the vector space semantics, a path is modeled as function Θ from the unit interval $[0,1]$ to vectors, i.e., a sequence of vectors. The domain of paths is written as Path.[4] For example,



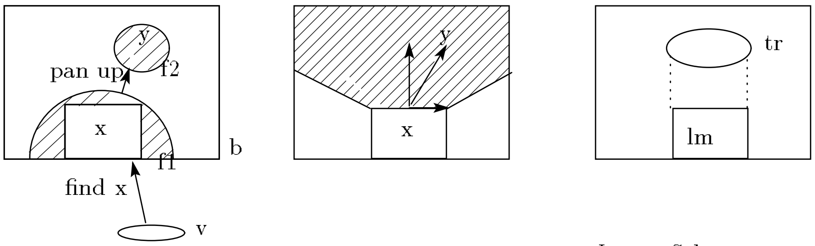
Private Viewpoint Model   Image Schema   Vector Space Semantics

Fig. 3. 'Over'

(3a) is representend as (3b) in the Vector Space Semantics, and as action (3c) in $DLPV$ actions, and as the image schemta in Fig 3.

(3) a. The bird flew over the house.
  b. $closest(\Theta, the\_house') \wedge \exists i \in [0,1].\mathbf{above}(the\_house)(\Theta(i))$, where Θ is the flying path of the bird and the_house' is the vector assigned to entity "the house".
  c. $(\mathsf{find}\ house; x := \mathsf{s}; \mathsf{pan\ to}\ side(x); y := \mathsf{b}; \mathsf{find}\ bird; (\mathsf{while}\neg(\mathsf{s} \sqsubset otherSide(x))\ \mathsf{do\ go}\ right))$

3.4 Locational Prepositions



Private Viewpoint Model   Vector Space Semantics   Image Schema

Fig. 4. 'x is above y'

(4) a. A bird is above the tree.
  b. $\mathbf{v} = the\_bird' \wedge ext(\mathbf{v}, the\_house') \wedge c(up, \mathbf{v}) > |\mathbf{v}_{\perp up}|$
  c. $(\mathsf{find}\ house; \mathsf{f} := \mathsf{s}; (\mathsf{while}\ \neg bird(\mathsf{s})\ \mathsf{do}(\mathsf{pan}\ up; \mathsf{s} :=?_{E(\mathsf{f})})))$

[4] $\Theta \in \mathsf{Path}$ is a *closest* path to a set of points $A \subseteq \mathsf{Point}$, written $closest(\Theta, A)$, iff every $i \in [0,1]$, $closest(\Theta(i), A)$.

3.5 Locational Prepositions and Cognitional Adjectives

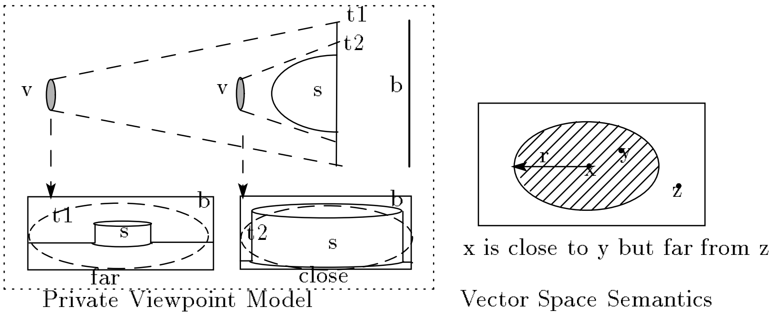


Fig. 5. 'Close' and 'Far'

As for locational prepositions 'close to' and 'far from', as in Fig. 5, $DLPV$ reflects the perspective from an agent, while the Vector Space Semantics does not. Similarly, cognitional use of adjectives like 'small' and 'big' can be formalized by the parallel way to 'far from' and 'close to', respectively, in $DLPV$. This property of $DLPV$ can be regarded as the reflection of agent's cognition of sizes and distances.

3.6 Motion Verbs

Asher and Sablayrolles [1] surveys and re-classifies motion verbs based on the following event structure:

$$\langle E, path(E), <, O, \bowtie, Goal, Source, Path, SIP, \{R\}_{R \in SP} \rangle,$$

where E is a set of events and other spatio-temporal objects, $<$ temporal precedence on E, O spatio-temporal overlap relation on E, $\bowtie$ spatio-temporal contact relation on E, $path(E) = E \cup (E \times O(E)) \cup (E \times O(E) \times O(O(E))) \cup \ldots \cup (E \times \ldots \times O^n(E))$ $(O(X) = \{e' | e, e' \in X, eOe'\})$, $Path : E \to path(E)$, $SIP : E \to pow(path(E))$ the "strict internal paths" of e (see [1]), $Goal, Source : path(E) \to E$, $SP = \{inner - halo, \bowtie_L, outer - halo, outer - most, inner - transit, contact - transit, outer - transit\}$ a set of spatial relations on E, and notions of locations, positions, and postures. They proposes four classes of motion verbs: verbs of change of location, verbs of change of position, verbs of inertial change of position, and verbs of change of posture; ten subclasses of verbs of change of location: Approach, Arrive, Enter, Land, Distance-from, Leave, Go-out, Take-off, Cross, Deviate.

In this section, I re-classify them (but not exhaustively) in terms of $DLPV$, i.e., instruction of movement of viewpoints , as the following table:

Motion	
□ Locational	$(x :=$ f; while f $= x$ do go)
□□ Bounded Locational	
□□□ Initially Bounded Locational	
□□□□ Leave	(leave from x) $\equiv$ (f $:= x$; while v $\bowtie$ f do go)
□□□□ Land	(land from x) $\equiv$ (f $:= x$; while v $\bowtie$ f do go up)
□□□ Finally Bounded Locational	(f $:= x$; while $\neg\varphi$(v, f) do go)
□□□□ Approach	(approach x) $\equiv$ (f $:= x$; while $\neg close$(v, f) do go)
□□□□ Arrive	(arrive x) $\equiv$ (f $:= x$; while $\neg$v $\bowtie$ f do go)
□□□□ Enter	(enter x) $\equiv$ (f $:= x$; while $\neg$v $\sqsubset$ f do go)
□□□ Fully Bounded Locational	
□□□□ Cross	(cross x) $\equiv$ (f $:= x$; pan to $side(x)$; while $\neg$s $\sqsubset$ $otherSide(x)$ do go $right$)
□□ Unbound Locational	
□□ Directed Unbound Locational	
□ Postural	
□□ Bounded Postural	
□□□ Finally Bounded Postural	(f $:= x$; while $\neg\varphi$(v, f) do go)
□□□□ Sit	(sit on x) $\equiv$ (f $:= buttoks$(s); f $:= y$; f $:= x$; while $\neg$f $\bowtie$ y do go $down$)
□□□□ Lay	(lay x) $\equiv$ (f $:= back$(s); f $:= y$; f $:= x$; while $\neg$f $\bowtie$ y do go $down$)
□□ Unbound Postural	
□□ Directed Unbound Postural	

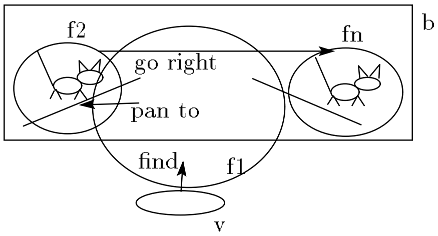


Fig. 6. 'Cross'

3.7 Dimensional Adjectives

Dimensional adjectives are a subgroup of *gradable adjectives*, that are those adjectives that order objects with respect to the relevant propery, e.g., *intelligent* and *big*. Dimensional adjectives' extent can be quantified using measure phrases such as *3cm long* and *20 years old* (See [6]). Faller [6] proposes the Vector Space Semantics of dimensional adjectives. For example, (5a) is interpreted as (5b) in the Vector Space Semantics, where $r_1 + r_2 < r_1$ means the direction of the vector is *negative*.

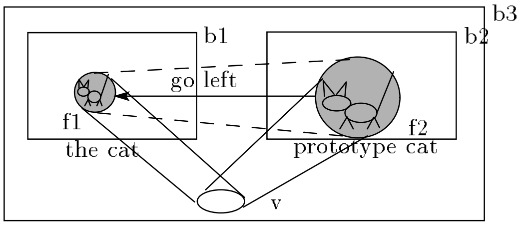


Fig. 7. 'Small'

In $DLPV$, dimensional adjectives can also be regarded as 'cognitive instructions', i.e., test command such that 'if the subject is compared with its prototype or *assumed/familiar degree* in the sense of coginitive psychology, both objects' degrees have some relation'. Firstly we introduce the following dimensional structure:

$$W = \langle dom(W), <_d \rangle_{d \in \mathsf{Degree}}$$

Secondly we introduce an invoking action of protypes from the memory into the background, as follows:

$$(\mathsf{prototype}\ \phi) \equiv ((\mathsf{while}\ \neg\phi(\mathsf{s})\ \mathsf{do}\ \mathsf{s} := ?_{E(memory)}); \mathsf{f} := \mathsf{s}; \mathsf{b} := \mathsf{b} \sqcup \mathsf{f})$$

Then, the test command is defined as in (5c) and illustrated as in Fig. 7.

(5) a. The cat is small.
  b. $\mathbf{v} = (r_1 \cdot u_s, r_2 \cdot u_s) \wedge r_1 + r_2 < r_2$, where u_s is a unit vector, i.e., $|u_s| = 1$, called a dimension vector of the size dimension, r is the extent of the size dimension of the cat.
  c. $(\mathsf{find}\ cat; [x := \mathsf{f}; \mathsf{prototype}\ cat; y := \mathsf{f}; (\mathsf{while}\ x \sqsubseteq y \vee y \sqsubseteq x\ \mathsf{do}; \mathsf{go}\ left)]x <_{size} y?)$

3.8 Cognitional Adjectives

Cognitional adjectives are not gradable adjectives and they reflect spatial cognition. Color, Shape, and Form such as *round*, *circular*, and *triangular* are typical cognitional adjectives. However, gradable adjectives can also have uses as cognitional adjectives. From a picture, we can recognize the vague information of 'depth', 'distance', 'size', 'width' and so on. As a matter of fact, in cognitive psychology, the degree notions of 'depth', 'distance', 'size', 'velocity' and so on are related to perception but not to real measuring. This supports the existence of cognitional uses of gradable adjectives such as 'deep/shallow', 'close/far', 'big/small', 'fast/slow' and so on.

Along this idea, test command '$A(x)$?' (x is A?) is definedd as follows:

$$A(x)? \equiv ([\mathsf{prototype}\ A; y := \mathsf{s}; \mathsf{find}\ x; similar(x, y)?]\top?)$$

if y is a cognitional adjective.

4 Dynamic Logic of Empathy

As we have seen in the introduction, a private cognition can be transfered to the other agents by language. In this process, the information on private viewpoints can also be transfered. Therefore, the receiver agents must adjust their viewpoints to the sender's viewpoint. This process can be regarded as an empathsing or sympathizing process, and by this process, agents can link privacy to publicity, then build community or society.

As a matter of fact, cognitive linguists find linguistic phenomena reflecting conveying the private viewpoint information, as follows.

4.1 Agent Resetting

(6) a. Interior perspective point: The lunchroom door opened and two men walked in.
   b. Exterior perspective point: Two men opened the lunchroom door and walked in. (two men's viewpoint?) ([36], p. 69)

(7) a. She sat in the rocker near her bed
   b. and looked out the window.
   c. How lovely the sky was! ([36], p. 68) ('Her' viewpoint)

(8) a. A man entered a room.
   b. The room was wide. ('the man's viewpoint')

By observing these examples, agentive subjects can reset the agent and the viewfinder to the agentive subjects and their viewfinders, while non-agentive subjects preserves the setting of the agent and the viewfinder. Such agent resettings are handled in $DLPV$ as simple action

$$(\mathsf{a} := x; \mathsf{v} := v(x))$$

4.2 Fictive Cognition

Cognitive linguistics enlighten us the following 'fictive motions' and 'fictive change'[5].

(9) a. The house is across the road.
   b. The scenery rushed past us as we drove along.
   c. This fence goes from the plateau to the valley.
   d. My shirt got smaller because I got fatter. (In Japanese, OK)
   e. The station got closer because a bridge was built. (In Japanese, OK)

[5] Eve Sweetser (2000) explains fictive changes in English using the theory of 'mental space'.

Congnitive linguists regard these examples as *subjective*, and explain these uses by introducing the notion of 'paths'. In other words, these uses are 'subjective' in terms of the setting of viewpoints and their changes. Also in $DLPV$, fictive motions and changes can be formalized as paths generated by while-loop of action $\mathsf{zoom\ in}$, $\mathsf{zoom\ out}$, and $\mathsf{pan}\ r$. The notion of 'path' in $DLPV$ can be defined as follows:

Definition 4. *A path of π from w is a set of finite sequences of states defined as follows:*

$$path(\pi, w) = \{path(\sigma, w) | \sigma \in SC(\pi)\}$$

$$path(\alpha, w) = \{\langle w, u\rangle | w \llbracket \alpha \rrbracket^{\mathcal{M}} u\}$$

$$path(\alpha\sigma, w) = \{\langle w, s\rangle | w \llbracket \alpha \rrbracket^{\mathcal{M}} u, s \in path(, \sigma, u)\}$$

where $SC(\pi)$ (finite computation sequences of π) is defined as follows:

$$\begin{aligned} SC(\alpha) &= \{\alpha\} \\ SC(\pi_1;\pi_2) &= SC(\pi_1)\cdot SC(\pi_2) \\ SC(\pi_1 \cup \pi_2) &= SC(\pi_1) \cup SC(\pi_2) \\ SC(\pi^*) &= \bigcup_{n\geq 0} SC(\pi^n) \end{aligned}$$

where $\pi^0 = \top?$, $\pi^{n+1} = \pi;\pi^n$, $S_1 \cdot S_2 = \{\sigma_1 \cdot \sigma_2 | \sigma_1 \in S_1, \sigma_2 \in S_2\}$ and $S \cdot \emptyset = \emptyset$.

4.3 Spatial Demonstratives

$$W = \langle dom(W), \mapsto, N, \bowtie_S\rangle$$

$w_1 \llbracket \mathsf{this} \rrbracket^{\mathcal{M}} w_2$ iff $\mathsf{f}_{w_2} N \mathsf{f}_{w_1}$
$w_1 \llbracket \mathsf{that} \rrbracket^{\mathcal{M}} w_2$ iff $\neg \mathsf{f}_{w_2} N \mathsf{f}_{w_1}$

$xNy \equiv \exists z.xN_z y.$

4.4 Tense and Aspect in Narratives

$$W = \langle dom(W), <_T, \sqsubseteq_T, \bowtie_T\rangle$$

$w_1 \llbracket \mathsf{past} \rrbracket^{\mathcal{M}} w_2$ iff $\mathsf{v}_{w_2} = \mathsf{v}_{w_1}$, $\mathsf{f}_{w_2} <_T \mathsf{f}_{w_1}$
$w_1 \llbracket \varphi \rrbracket^{\mathcal{M}} w_2$ iff if $\varphi \notin Stative$, then $\mathsf{f}_{w_2} \bowtie_T \mathsf{f}_{w_1}$ and $\mathsf{f}_{w_1} <_T \mathsf{f}_{w_2}$

5 Conclusion

In this paper, I have proposed logical system $DLPV$ that is an extension of First-Order Dynamic Logic and its model, the Private Viewpoint Model, that reflects agents' private cognition of their environments and the communication of the information of this private cognition. We have seen that some fragment of natural language can be handled as a set of complex actions on operation of the Private Viewpoint Model, that is a formal realization of the idea "language-as-cognitive-instruction". The total system that I have proposed can be regarded as a formalization of "ecological" realism, that is compatible with the Gibsonian ecological psychology rather than the Gestalt psychology that is compatible with "cognitive linguistics", and more "cognitive" than formal ontology such as "Vector space semantics".

The investigation of the logical properties of $DLPV$ and its comparison with the Vector Space Semantics and other axiomatic systems based on "naive" space ontologies are my future work.

Appendix: Part of Axioms of *DLPV*

Axioms and Inference Rules of DL
$\forall x[x :=?_X]\varphi \equiv (\forall x \in X.\varphi)$
$\forall x[x := \mathsf{v}][\mathsf{zoom\ in}]\mathsf{v}N_{\mathsf{x}}\mathsf{f}$
$\forall x[x := \mathsf{f}][\mathsf{come}]\mathsf{f}N_{\mathsf{x}}\mathsf{v}$
$\forall x[x := \mathsf{f}][\mathsf{pan}\ r]\mathsf{f}r\mathsf{x}$
$\forall x[x := \mathsf{f}][\mathsf{pan}\ ?]\exists r.\mathsf{f}r\mathsf{x}$
$\forall x.\neg x \mapsto x$
$\forall xyz(x \mapsto y \wedge y \mapsto z) \to x \mapsto z$
$\forall x.x \sqsubseteq x$
$\forall xyz(x \sqsubseteq y \wedge y \sqsubseteq z) \to x \sqsubseteq z$
$\forall xy(x \sqsubseteq y \wedge y \sqsubseteq x) \to x = z$
$\forall xyz.xN_y z \to x \neq y \neq z$
$\forall xyz.xN_y z \to xN_z y$
$\forall xyz.xN_y z \to \neg yN_x z$
$\forall xyzu(xN_y z \wedge uN_x z) \to xN_y u$
$\forall xyzu(xN_y z \wedge zN_x u) \to xN_y u$

For axioms on N, see the axioms of 'betweenness' of [8].

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