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多目的ナース・リスケジューリング問題における平等性

Egalitarianism in Multi-Objective Nurse Rerostering Problem

呉 詩敏 [ϒ] Shih-Min Wu
沖本 天太 [ϒ] Tenda Okimoto
平山 勝敏 [ϒ] Katsutoshi Hirayama
井上 克巳 [†] Katsumi Inoue

[ϒ] 神戸大学大学院海事科学研究科 Faculty of Maritime Sciences, Kobe University
[†] 国立情報学研究所 National Institute of Informatics

How to schedule a limited number of nurses in hospital wards staffed 24 hours a day is important issue for the satisfactory patient care and potentially improve nurse retention. Nurse Scheduling Problem (NSP) is a combinatorial optimization problem, in which a set of nurses must be assigned into a limited set of working slots, subject to a given set of hard and soft constraints. It is natural to consider the scheduled nurse's unexpected absences. Nurse Rerostering Problme (NRP) is a dynamic NSP where the aim is to reschedule the current roster so that the number of changes of assignments between current and modified schedules is minimized. In this paper, the focus is laid on NRP with multiple criteria and the egalitarianism among nurses in a modified schedule. A formal framework for Multi-Objective NRP (MO-NRP) is provided and a novel solution criterion (egalitarianism) for MO-NRP is defined.

1. Introduction

Nurse Scheduling Problem (NSP) [1, 2, 13, 14] is one of the representative application problems in OR and AI. This problem can be represented as an Weighted CSP [1] where the aim is to find an assignment that satisfies all hard constraints and minimizes the sum of all violated costs of soft constraints. In order to provide satisfactory patient care and potentially improve nurse retention, creating a good schedule for nurses is important issue. However, in general, since making the schedule which satisfies all constraints is intractable, the scheduler spends a lot of time to find an acceptable schedule for both nurses and the hospital. In NSP, various complete and incomplete approaches have been introduced to generate better nurse schedules [2, 3, 13, 14].

Nurse Rerostering Problme (NRP) [6, 9, 11, 16] is a dynamic NSP where the aim is to reschedule the current roster so that the number of changes of assignments (shift works) between current and modified schedules is minimized. It is natural to consider the scheduled nurse's unexpected absences, e.g., illness, accident and injury. When an absence is announced, the scheduler must find a nurse who can fill the vacancy of the absentee and the current schedule must be rebuilt as soon as possible. Most previous works for NRP focus on the *stability* of a schedule, i.e., the new schedule should be similar to the current one as much as possible.

The *egalitarianism* is an expected property of an NRP. Even if the number of changes of all assignments in a modified schedule is small (and it is also optimal, i.e., all hard constraints are satisfied and the sum of the violation costs of soft constraints is minimized), it can happen that one needs to change her assignments a lot, while others not.

In this paper, the focus is laid on NRP with multiple criteria and the egalitarianism among nurses. A novel framework for *Multi-Objective Nurse Rerostering Problem* (MO-NRP) is introduced which is the extension of an NRP. More specifically, MO-NRP is modeled by using the framework of Multi-Objective Weighted CSP (MO-WCSP) [10] which is the extension of Weighted CSP [5] where the aim is to find an assignment that satisfies all hard constraints and minimizes all objectives simultaneously. Solving an MO-NRP is to find Pareto optimal (i.e. trade-off) solutions among "optimality" and "stability". Also, a novel solution criterion called egalitarian solution for MO-NRP is defined.

NRP is an application problem of a Minimal Perturbation Problem (MPP) [4, 12] which is a problem for Dynamic CSP where the aim is to find a valid solution that minimizes a given distance function. Usually, the distance function measures the number of changing variables. Minimizing perturbations then results in minimizing the number of changes in the assignment. Solving an MPP is finding a stable solution. Compared to MPP, this work focuses on multiple criteria, i.e., optimality and stability of a schedule.

Hattori et al. [15] formalized a dynamic NSP by using dynamic weighted MaxCSP which can effectively deal with dynamic changes to a problem. They introduced provisional constraints which allow variables to keep the same values so that one can obtain a stable solution that is close to previous ones. This paper works on MO-NRP and focuses on the egalitarianism among nurses, while they worked on the stability in dynamic (mono-objective) NSP (i.e. NRP).

Pato et al. [11] worked on bi-objective genetic heuristic for NRP which considers to minimize the sum of the changes of assignments and the number of constraint violations like classical NSP. Also, Burke et al. [3] investigated multiple criteria in NSP. Compared to these works, this paper focuses on the egalitarianism among nurses of a schedule.

The rest of the paper is organized as follows. In the next section, the formalizations of NRP and MO-WCSP are provided. Afterwards, our framework for MO-WCSP based MO-NRP is presented and the formal definition of a egalitarian solution for an MO-NRP is provided. Finally, we conclude this paper and give some future works.

連絡先: Shih-Min Wu, Graduate School of Maritime Sciences, Kobe University, 148w402w@stu.kobe-u.ac.jp

2. Preliminaries

The formalizations of a Nurse Rerostering Problem and a Multi-Objective Weighted CSP are briefly described.

Nurse Rerostering Problem

Nurse Rerostering Problem (NRP) [6, 9, 11, 16] is a combinatorial optimization problem, in which a set of nurses must be assigned into a limited set of working slots, subject to a given set of hard and soft constraints. In general, the constraints are dependent on the requirements of both nurses and hospitals. The following is the representative hard and soft constraints, which are used in previous works.

HC 1 : Prohibited working patterns must be avoided (e.g. 7 consecutive works and 3 consecutive night shifts).

HC 2 : There exists the required number of nurses for each shift in a day, e.g., at least 3 nurses must be assigned to the morning shift and 2 for evening and night shifts.

HC 3 : The number of day-offs of each nurse must be same before and after the modification.

HC 4 : Each newcomer should be assigned together with skillful nurse (i.e., head or highly experienced nurse).

HC 5 : Nurses must rest at least 16 hours between two consecutive shift works, e.g., a morning shift (8:00-16:00) and an evening shift (16:00-24:00) should not be assigned after a night shift (0:00-8:00).

S1 : For each shift work, the required skill level of assigned nurses should be satisfied.

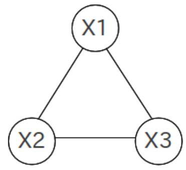
S2 : Day-offs of each nurse in a current schedule should not be changed in a modified schedule.

S4 : Requests of nurses (e.g. preferred working pattern and specially day-off request) should be satisfied as much as possible.

Objective : Minimize the number of changes of shift works between current and modified schedules.

Multi-Objective Weighted CSP

Multi-Objective Weighted CSP (MO-WCSP) [10] is the extension of Weighted CSP [5] where the aim is to find an assignment that satisfies all hard constraints and minimizes the sum of all violated costs of soft constraints. Let k be the number of objectives. MO-WCSP is defined by a tuple $MO\text{-}WCSP = \langle X, D, C, S, \Phi\rangle$, where $X = \{x_1, x_2, ..., x_n\}$ is a set of variables, $D = \{d_1, d_2, ..., d_m\}$ is a set of domains, $C = \{C^1, C^2, ..., C^k\}$ is a set of hard and soft constraints, $S = \{S^1, S^2, ..., S^k\}$ is a set of valuation structures, $\Phi = \{\phi^1, \phi^2, ..., \phi^k\}$ is a set of multi-objective functions. For each objective i $(1 \leq i \leq k)$, $C^i = C^i_h \cup C^i_s$ is the union of hard and soft constraints, where C^i_h is a set of hard constraints and C^i_s is a set of soft constraints, $S^i = (E^i, \sum, <)$ is the valuation structure, where $E^i = \mathbb{N} \cup \{\infty\}$, $\sum$ is the standard sum over $\mathbb{N}$ and all elements of E are ordered by the operator $<$, and $\phi^i : C^i \to E^i$ is a cost function. Let



x_1	x_2	cost
0	0	(∞,0)
0	1	(0,1)
1	0	(∞,1)
1	1	(∞,0)

x_1	x_3	cost
0	0	(3,1)
0	1	(0,3)
1	0	(3,2)
1	1	(2,1)

x_2	x_3	cost
0	0	(1,3)
0	1	(2,∞)
1	0	(0,4)
1	1	(4,1)

図 1: Example of bi-objective WCSP.

A be an assignment to all variables. For an objective i, the valuation of A for constraint $c \in C^i$ is defined as:

$$\phi^i(A, c) = \begin{cases} 0 & c \in C^i_h \text{ } is\ satisfied\ by\ A, \\ \infty & c \in C^i_h \text{ } is\ violated\ by\ A, \\ \phi^i(A, c) & c \in C^i_s. \end{cases}$$

and the overall valuation of A is given by

$$\phi^i(A) = \sum_{c \in C^i} \phi^i(A, c).$$

Then, the sum of the violation costs of all cost functions for k objectives is defined by a cost vector, denoted $\Phi(A)$ $= (\phi^1(A), \phi^2(A), ..., \phi^k(A))$. Finding an assignment that minimizes all objective functions simultaneously is ideal. However, in general, since trade-offs exist among objectives, there does not exist such an ideal assignment. Therefore, the "optimal" solution of an MO-WCSP is characterized by using the concept of *Pareto optimality*. MO-WCSP can be represented using a constraint graph, in which a node corresponds to a variable and an edge represents a constraint.

Definition 1 (Dominance)**.** For two cost vectors $\Phi(A)$ and $\Phi(A')$, we call that $\Phi(A)$ dominates $\Phi(A')$, denoted by $\Phi(A) \prec \Phi(A')$, iff $\Phi(A)$ is partially less than $\Phi(A')$, i.e., it holds (i) $\phi^i(A) \leq \phi^i(A')$ for all objectives i, and (ii) there exists at least one objective i', such that $\phi^{i'}(A) < \phi^{i'}(A')$.

Definition 2 (Pareto optimal solution)**.** An assignment A is said to be the Pareto optimal solution, iff there does not exist another assignment A', such that $\Phi(A') \prec \Phi(A)$.

Definition 3 (Pareto Front)**.** A set of cost vectors obtained by Pareto optimal solutions is said to be the Pareto front. Solving an MO-WCSP is to find the Pareto front.

Example 1 (MO-WCSP)**.** Consider the complete graph of a bi-objective WCSP with three variables x_1, x_2 and x_3. Figure 1 shows the cost vectors among three variables. Each variable takes its value from $\{0, 1\}$. Each table show the cost vector for each constraint. For example, for the constraint between x_1 and x_3, in case x_1 and x_3 take same value 0, the obtained cost vector is $(3, 1)$, i.e., cost 3 for objective 1 and cost 1 for objective 2. The cost ∞ means that it violates a hard constraint. The Pareto optimal solutions of this problem are $\{\{(x_1, 0), (x_2, 1), (x_3, 0)\}, \{(x_1, 0), (x_2, 1), (x_3, 1)\}\}$ and the obtained Pareto front is $\{(3, 6), (4, 5)\}$.

表 1: Example of $MS_{current}$ for a week of 7 nurses.

Nurse（Lebel）	M	T	W	T	F	S	S
n_1 (l_1)	o	m	m	m	e	e	e
n_2 (l_2)	e	e	n	o	m	m	m
n_3 (l_3)	m	m	m	e	e	n	o
n_4 (l_3)	m	e	e	n	o	m	n
n_5 (l_4)	m	m	e	e	n	o	m
n_6 (l_4)	n	n	o	m	m	e	e
n_7 (l_5)	e	o	m	m	m	m	m

3. Multi-Objective NRP

In order to consider the minimizing the number of (i) constraint violations (optimality) and (ii) the changes of assignments (stability) simultaneously in an NRP, a Multi-Objective Nurse Rerostering Problem (MO-NRP) is formalized by using the framework of an MO-WCSP. Moreover, the egalitarian solution for an MO-NRP is defined.

Let us describe the following basic terms for MO-NRP.

- $N = \{1, ..., n\}$ is a set of ID-numbers for nurses.
- $M = \{1, ..., m\}$ is a set of days in a scheduling period.
- $X = \{x_{11}, ..., x_{nm}\}$ is a set of variables.
- $W = \{o, m, e, n\}$ is a set of shift works, where $o = \{\text{day-off}\}$, $m = \{\text{morning}\}$ (8:00-16:00), $e = \{\text{evening}\}$ (16:00-24:00) and $n = \{\text{night}\}$ (0:00-8:00).
- $L = \{l_1, ..., l_5\}$ is a set of skill levels of nurses where $l_1 = \{\textit{head nurse}\}$, $l_2 = \{\textit{highly experienced}\}$, $l_3 = \{\textit{experienced}\}$, $l_4 = \{\textit{few years}\}$ and $l_5 = \{\textit{newcomer}\}$.
- $\alpha_l : N \to L$ is a mapping from N to L which provides the skill level of a nurse, e.g., for a head nurse $i \in N$, her skill level can be obtained by $\alpha_l(i) = l_1$.

A $(n \times m)$-table is said to be a master schedule and is denote as $MS_{current}$ for the current schedule and MS_{mod} for the modified schedule after unexpected absences of a nurse. $MS_{current}$ is a solution of NSP and MS_{mod} is that of NRP.

Definition 4 (Stability)**.** For $MS_{current}$ and MS_{mod}, each $w_{ij} \in W$ in $MS_{current}$ and each $w'_{ij} \in W'$ in MS_{mod}, and a non-negative integer r, MS_{mod} is said to be *r-stable*, iff the sum of the changes of assignments is bounded by r, i.e.,

$$\sum_{i,j} g(w_{ij}, w'_{ij}) \leq r, \ \textit{where} \ g(w_{ij}, w'_{ij}) = \begin{cases} 0 & w_{ij} = w'_{ij}, \\ 1 & \textit{otherwise}. \end{cases}$$

Example 2. Consider a muster schedule for a week of 7 nurses. Table 1 represents the current master schedule $MS_{current}$ which satisfies all hard constraints (HC 1 - HC 5 in section 2). Assume that nurse n_5 has an unexpected absence on Monday and cannot work her shift work, i.e., morning shift m. Table 2 shows a modified muster schedule MS_{mod}. The morning shift of n_5 on Monday has been changed from m to absence in MS_{mod} (denoted by /). From HC 2 (i.e. at least 3 nurses must be assigned to the morning shift and 2 for evening and night shifts), nurse n_1 works the shift work m instead of n_5. In order to satisfy all hard constraints (from HC 1 to HC 5), nurse n_1 changes her shift works (i.e. evening shifts e) on Friday, Saturday and Sunday in $MS_{current}$ to night shift n on Friday, day-off o on Saturday and morning shift m on Sunday in MS_{mod}. Also, nurse n_5 changes her night shift n on Friday, day-off o on Saturday and morning shift m on Sunday in $MS_{current}$ to evening shifts e on these three days in MS_{mod}. Since the number of changes is 8 (including the absence of n_5 on Monday), the modified schedule MS_{mod} is r = 8-stable.

表 2: Example of a modified schedule MS_{mod}. Nurse n_5 had an unexpected absence on Monday (denoted by /). Red fonts show the modified shift works.

Nurse（Lebel）	M	T	W	T	F	S	S
n_1 (l_1)	$\mathbf{m}$	m	m	m	$\mathbf{n}$	$\mathbf{o}$	$\mathbf{m}$
n_2 (l_2)	e	e	n	o	m	m	m
n_3 (l_3)	m	m	m	e	e	n	o
n_4 (l_3)	m	e	e	n	o	m	n
n_5 (l_4)	/	m	e	e	$\mathbf{e}$	$\mathbf{e}$	$\mathbf{e}$
n_6 (l_4)	n	n	o	m	m	e	e
n_7 (l_5)	e	o	m	m	m	m	m

In the following, the framework for MO-NRP is defined.

Definition 5 (MO-NRP)**.** An MO-NRP is a tuple MO-NRP $= \langle X, W, L, C, S, MS_{current}, \Phi \rangle$, where X is a set of variables, W is a set of domains, L is a set of skill levels, C and S are same as MO-WCSP, $MS_{current}$ is the current schedule, $\Phi = \{\phi^{opt}, \phi^{stable}\}$ is a set of cost functions where ϕ^{opt} is a cost function for optimality and ϕ^{stable} is that for stability. For a value assignment A to all variables, the sum of the violation costs of all cost functions and the sum of the changes of assignments is given by a vector $\Phi(A) = (\phi^{opt}(A), \phi^{stable}(A))$. Solving an MO-NRP is to find Pareto optimal solutions so that (i) all hard constraints are satisfied, (ii) the sum of the violation costs and (iii) the sum of the changes of assignments are minimized.

In previous works on MO-NRP, the aim is to find an assignment so that the number of the changes of assignments between current and modified schedules is minimized, i.e., stability. On the other hand, in MO-NRP, bi-objectives (i.e. optimality and stability) are considered simultaneously.

In MO-NRP, one can easily define several objective functions (i.e. ϕ^{opt_1}, ϕ^{opt_2},...,ϕ^{opt_p}) instead of only one objective function ϕ^{opt}. For the simplicity, this paper defines ϕ^{opt} for optimality like classic NSP. Such simplification can be done by aggregating all objective functions called AOF technique [7] (or in other words, scalarization method).

Definition 6 (s-vector)**.** Let $s_i = \sum_j g(w_{ij}, w'_{ij})$ be the number of the changes of assignments for a nurse i. The number of the changes of assignments for all nurses is said to be a *s-vector w.r.t. MS* and denoted by $v_s = (s_1, ..., s_n)$.

表 3: Example of a modified schedule MS'_{mod} which is more egalitarian than MS_{mod}.

Nurse（Lebel）	M	T	W	T	F	S	S
n_1 (l_1)	**m**	m	m	m	e	**o**	e
n_2 (l_2)	e	e	n	o	m	m	m
n_3 (l_3)	m	m	m	e	**n**	n	o
n_4 (l_3)	m	e	e	n	o	m	**m**
n_5 (l_4)	/	m	e	e	**e**	**e**	**n**
n_6 (l_4)	n	n	o	m	m	e	e
n_7 (l_5)	e	o	m	m	m	m	m

Definition 7 (Equivalence)**.** For two s-vectors $v_s = (s_1, ..., s_n)$ and $v_{s'} = (s'_1, ..., s'_n)$ w.r.t. MS_{mod}, v_s and $v_{s'}$ are said to be *equivalent*, iff it holds $\sum_{i=0}^{n} s_i = \sum_{i=0}^{n} s'_i$.

Let V_s be a set of equivalent s-vectors w.r.t. MS_{mod} and $\preceq_{lex}$ be the total preoder over V_s defined $\forall v_s, v_{s'} \in V_s$ as $v_s \preceq_{lex} v_{s'}$ if and only if lexically reordered v_s precedes lexically reordered $v_{s'}$. Let $v_s = (4, 1, 3, 2, 2)$ and $v_{s'} = (4, 0, 3, 2, 3)$ be two vectors. The corresponding reordered vectors are $\overline{v_s} = (4, 3, 2, 2, 1)$ and $\overline{v_{s'}} = (4, 3, 3, 2, 0)$. Compare the 1st components of $\overline{v_s}$ and $\overline{v_{s'}}$. In case they are same, the 2nd components are compared. Continue to compare until one of two components is smaller than the another one. In this example, for the 3rd components, since 2 of $\overline{v_s}$ is smaller than 3 of $\overline{v'_s}$, v_s is lexically smaller than v'_s.

Definition 8 (Egalitarianism)**.** For an MO-NRP, a MS_{mod} and a s-vector w.r.t. MS_{mod}, v_s is said to be a egalitarian solution of MS_{mod}, iff there does not exist another equivalent s-vector $v_{s'}$ w.r.t. MS_{mod}, such that $v_{s'} \preceq_{lex} v_s$.

Example 3. Consider the MS_{mod} in table 2. The s-vector w.r.t. MS_{mod} is $v_s = (4, 0, 0, 0, 4, 0, 0)$. Table 3 shows the alternative muster schedule MS'_{mod}. Since the number of the changes of assignments is 8, MS'_{mod} is r = 8-stable and the s-vector w.r.t. MS'_{mod} is $v_{s'} = (2, 0, 1, 1, 4, 0, 0)$, i.e., v_s and $v_{s'}$ are equivalent. The lexically reordered vectors of v_s and $v_{s'}$ are $\overline{v_s} = (4, 4, 0, 0, 0, 0, 0)$ and $\overline{v_{s'}} = (4, 2, 1, 1, 0, 0, 0)$. Thus, $v_{s'}$ is more egalitarian than v_s ($v_{s'} \preceq v_s$). Compared to MS_{mod}, five nurses (i.e. n_1, n_3, n_4 and n_5) share the changes in MS'_{mod}, while only two nurses (i.e. n_1 and n_5) changes their assignments in MS_{mod}.

4. Conclusion

NRP is a dynamic NSP where the aim is to reschedule the current roster so that the number of the changes of assignments between current and modified schedules is minimized. Most previous works on NRP focused on the stability, i.e., the new schedule should be similar to the current one as much as possible. The contribution of this paper is twofold:

- A formal framework of Multi-Objective Nurse Rerostering Problem (MO-NRP) is defined. The aim of an MO-NRP is to find the trade-off solutions among "optimality" and "stability" of the modified schedule.
- The "egalitarianism" among nurses has been first studied in MO-NRP.

As a perspective for further research, we intend to apply our approach to some real problems and analyze the trade-off solutions for an MO-NRP. Furthermore, we will develop an efficient algorithm for solving an MO-NRP. Also, we are interested in time tabling problems, e.g., educational, sport, transportation and entertainment time tables [8].

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